Atmospheric Physics I

- 2f. Determine the relationships between fundamental atmospheric variables.

Equation of State

- This equation is introduced to show the interdependency of pressure, density and temperature for the atmosphere.
- $P = \rho R T$
  - P is the atmospheric pressure.
  - ρ is the atmospheric density. ($\rho = m / V$)
  - R is the gas constant for dry air (always constant).
  - T is the temperature of dry air, in Kelvin (for moist air, the virtual temperature (T_v) is used in the equation of state).

Equation of State

- Variable Relationships
  - At a constant temperature, pressure and density are directly proportional $\uparrow \dot{T} = \uparrow \rho R$.
  - At a constant pressure, density and temperature are inversely proportional $\dot{P} = \uparrow \rho R \downarrow T$.
  - At constant density, pressure and temperature are directly proportional $\dot{\rho} \uparrow P = R \uparrow T$.

Equation of State

- Review of density dependence on mass and volume.
  - Density is inversely proportional to volume, with mass held constant.

$$\uparrow \rho = \frac{\dot{m}}{\downarrow V_{ol}}$$

  - Temperature and volume are directly proportional, with all other variables held constant.

$$\dot{P} = \frac{\dot{m}\ \dot{R}}{\downarrow V_{ol}} \quad \downarrow T$$

Equation of State

- Pressure and volume are inversely proportional, with all other variables held constant.

$$\uparrow P = \frac{\dot{m}\ \dot{R}\ \dot{T}}{\downarrow V_{ol}}$$

- Density is directly proportional to mass, with volume held constant.

$$\uparrow \rho = \frac{\uparrow m}{\dot{V}_{ol}}$$

Equation of State

- Temperature and mass are inversely proportional, with all other variables held constant

$$\dot{P} = \uparrow \frac{\dot{m}\dot{R}}{\dot{V}_{ol}} \quad \downarrow T$$

- Pressure and mass are directly proportional, with all other variables held constant.

$$\uparrow P = \uparrow \frac{\dot{m}\dot{R}\,\dot{T}}{\dot{V}_{ol}}$$

Hypsometric Equation

- The thickness of a layer is proportional to the mean virtual temperature for the layer. This layer is bounded vertically by pressure (isobaric) surfaces (i.e. 1000-500 mb thickness would be the layer bounded by 1000 mb and 500 mb pressure surfaces).
- $\Delta Z = (kR/g)\overline{T_v}$

$$\Delta Z = (kR/g)\overline{T}_v$$

- Variables Defined
  - ∀ ΔZ is the thickness of a layer.
  - k is a calculus based ratio of upper and lower pressure surfaces (i.e. the natural logarithm of P_{upper} / P_{lower})
  - $\overline{T}_v$ is the mean virtual temperature for the layer.
  - Recall that $\overline{T}_v$ = T + w/6
  - Virtual temperature takes into account both the temperature and the moisture content of the air.
  - R is the gas constant for dry air.
  - g is the gravitational force (assumed to be constant).

Hypsometric Equation

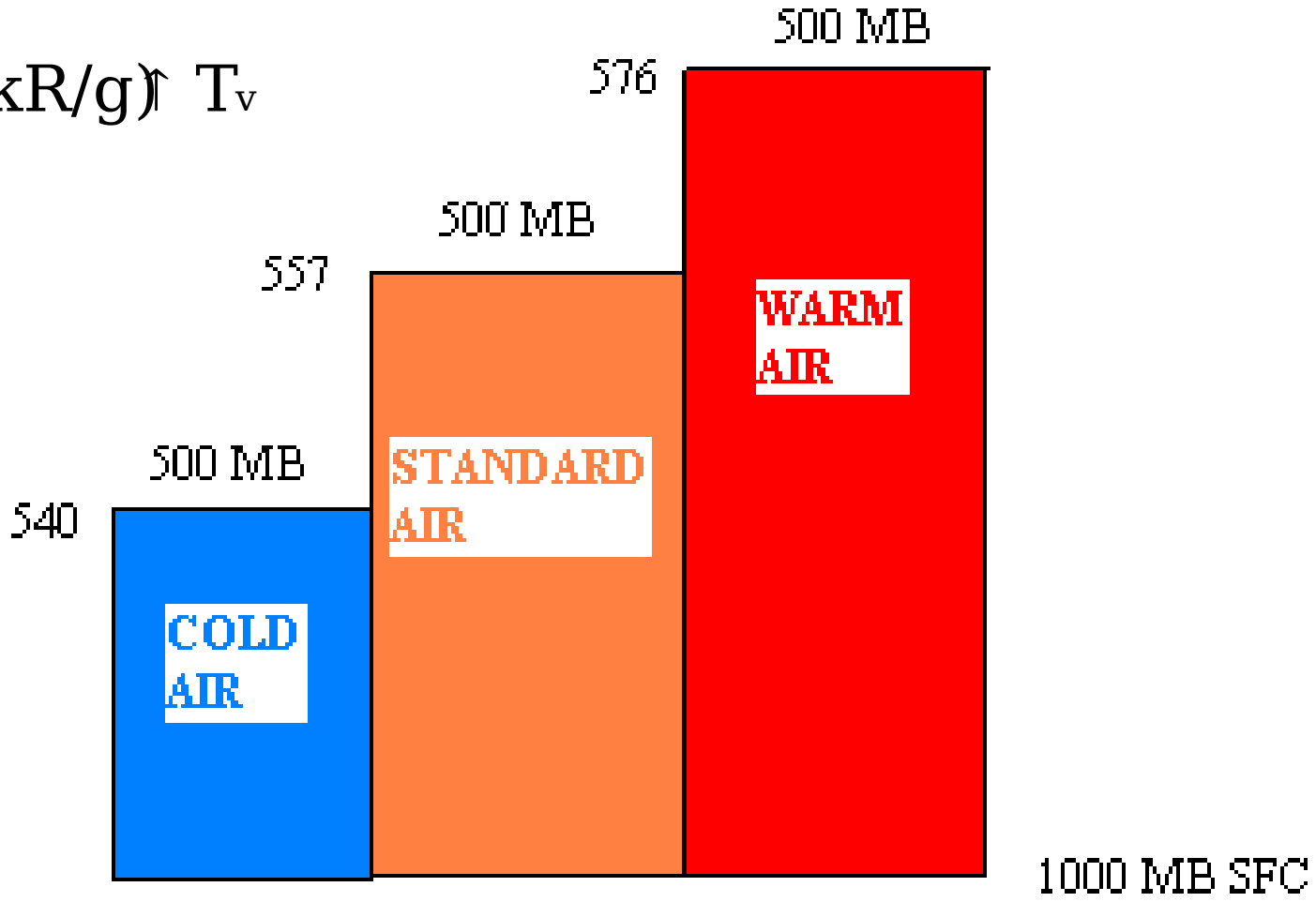
- Variable Relationships
  - Thickness (ΔZ) is directly proportional to the mean virtual temperature ($\overline{T_v}$) of the layer.
  - The thickness of a layer is greater for a warm column of air than for a cold column.

$$\uparrow \Delta Z = (kR/g)\,\overline{T_v}\uparrow$$

  - There is more space between molecules of warm air than for cold air; thus, a given mass of warm air occupies more verti cal space (i.e. has higher thickness) than an equivalent mass of cold air (i.e. has lower thickness)

$$\uparrow T_v = \uparrow T + \frac{w}{6}$$

$$\uparrow \Delta Z = (kR/g) \uparrow T_v$$



Hypsometric Equation

- Thickness is directly proportional to temperature.
- As the temperature increases, the virtual temperature increases and therefore the thickness increases.

$$\uparrow T_v = \uparrow T + w/6$$

$$\uparrow \Delta Z = k \uparrow T_v$$

Therefore, $\Delta Z \propto T$

Hypsometric Equation

- Thickness is directly proportional to the mixing ratio (w).
- As the mixing ratio increases, the virtual temperature increases and therefore the thickness increases.

  $\uparrow T_v = T + \uparrow w/6$

  $\uparrow \Delta Z = (kr/G) \uparrow T_v$

  Therefore, $\Delta Z \propto w$

Hypsometric Equation

- NOTE: MEAN LAYER TEMPERATURE HAS A MUCH LARGER EFFECT ON THICKNESS THAN DOES MEAN MIXING RATIO.

Hypsometric Equation

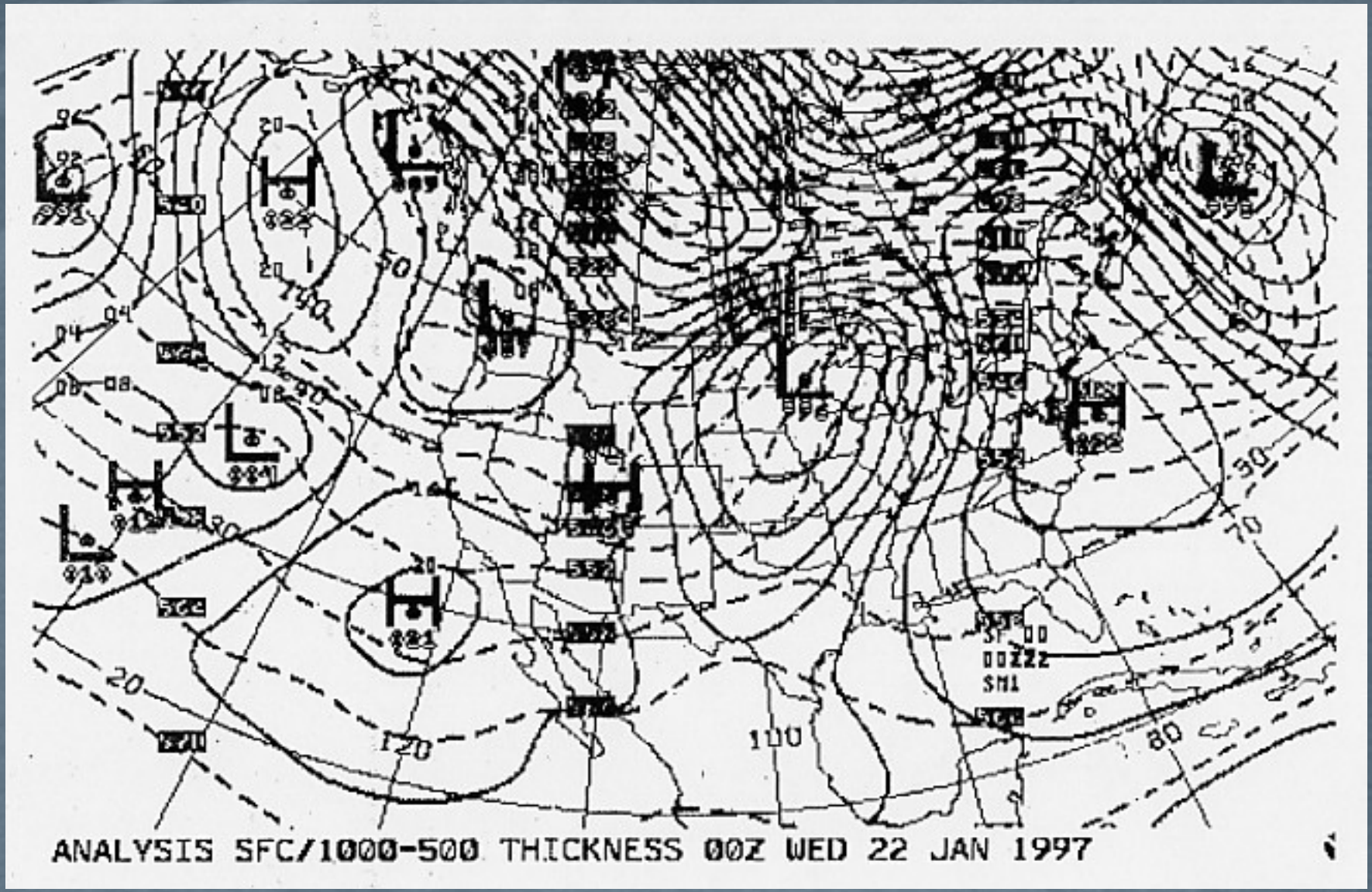
- Examples
  - Between 850 mb and 700 mb, strong cold air advection is occurring. How does the thickness of the layer change? (Answer: temperature decreases, therefore THICKNESS DECREASES.)

    $\downarrow Z = (kR/g) \downarrow T_v$
  - Moist air advection occurs between 1000 and 850 mb. What is happening to the mixing ratio? Has the thickness increased or decreased? (Answer: mixing ratio increases, therefore THICKNESS INCREASES.)

    $\uparrow Z = (kR/g) (T + \uparrow w/6)$

Hypsometric Equation

- Atmospheric Significance
- In general, lower thickness values are seen at the higher latitudes (where the temperature is colder).
- The 1000-500 mb thickness chart is used to evaluate temperature advection (more in a later objective).


ANALYSIS SFC/1000-500 THICKNESS 00Z WED 22 JAN 1997

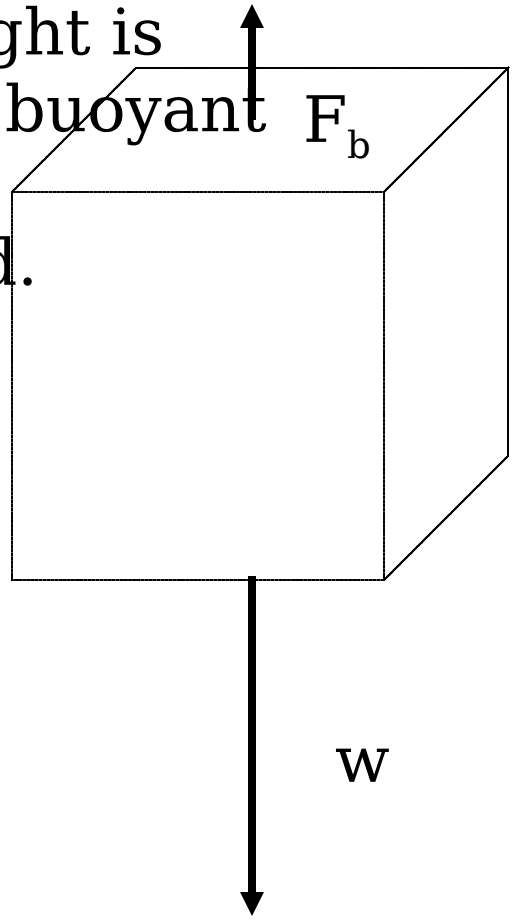
Atmospheric Stability

- Archimedes principle
  - When a parcel of air is displaced in the atmosphere, the atmosphere exerts a force upward on the parcel equal to the weight of the atmosphere displaced.
  - This force is called the BUOYANT FORCE.

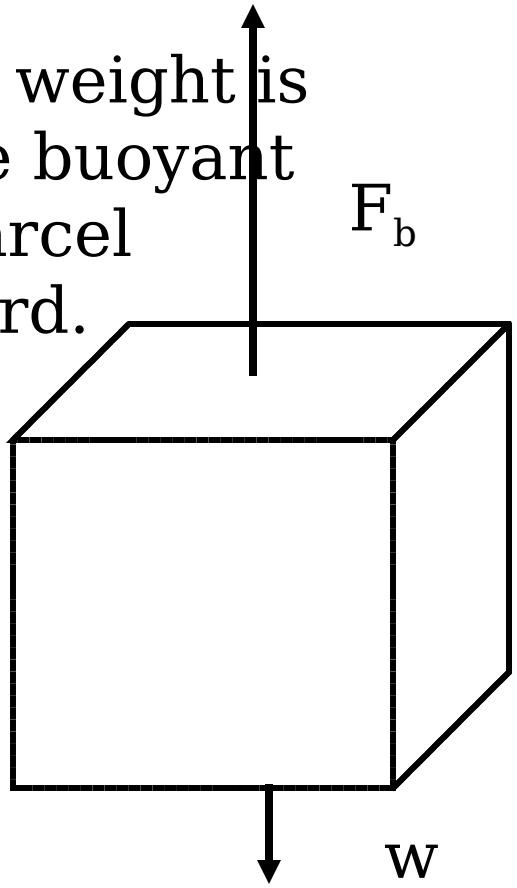
Atmospheric Stability

- Archimedes principle
  - Comparison of the forces acting on a parcel.
    - Weight of the parcel (w) - always acts toward the ground.
    - Buoyant force (F_b) - always acts in the opposite direction of the weight.
    - If the parcel weight is greater than the buoyant force, the parcel moves DOWNWARD.
    - If the parcel weight is less than the buoyant force, the parcel moves UPWARD.
    - If the parcel weight is equal to the buoyant force, the parcel remains STATIONARY. This is known as neutral buoyancy.

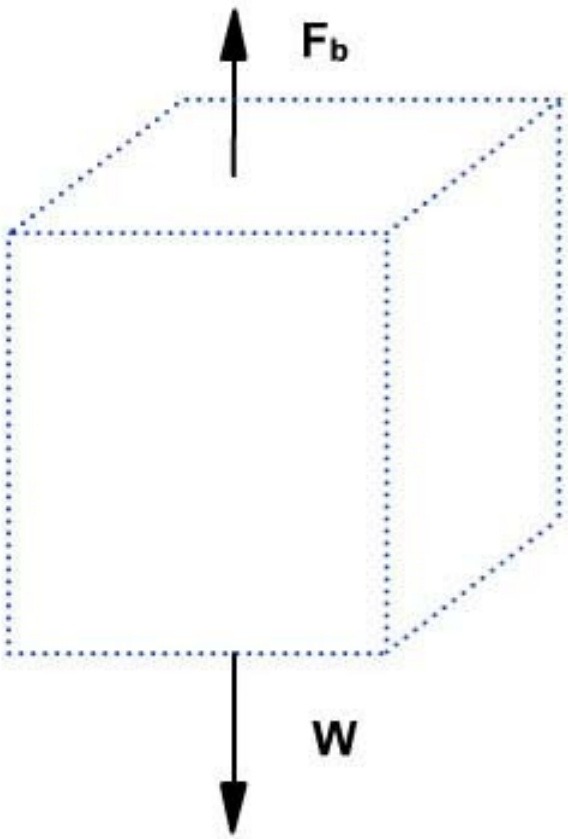
If the parcel weight is greater than the buoyant force, the parcel moves downward.



If the parcel weight is less than the buoyant force, the parcel moves upward.



Evaluating Bouyant Force on an Air Parcel



(bouyant force)

If the parcel weight is equal to the buoyant force, the parcel remains STATIONARY.

(weight of the air parcel)

Atmospheric Stability

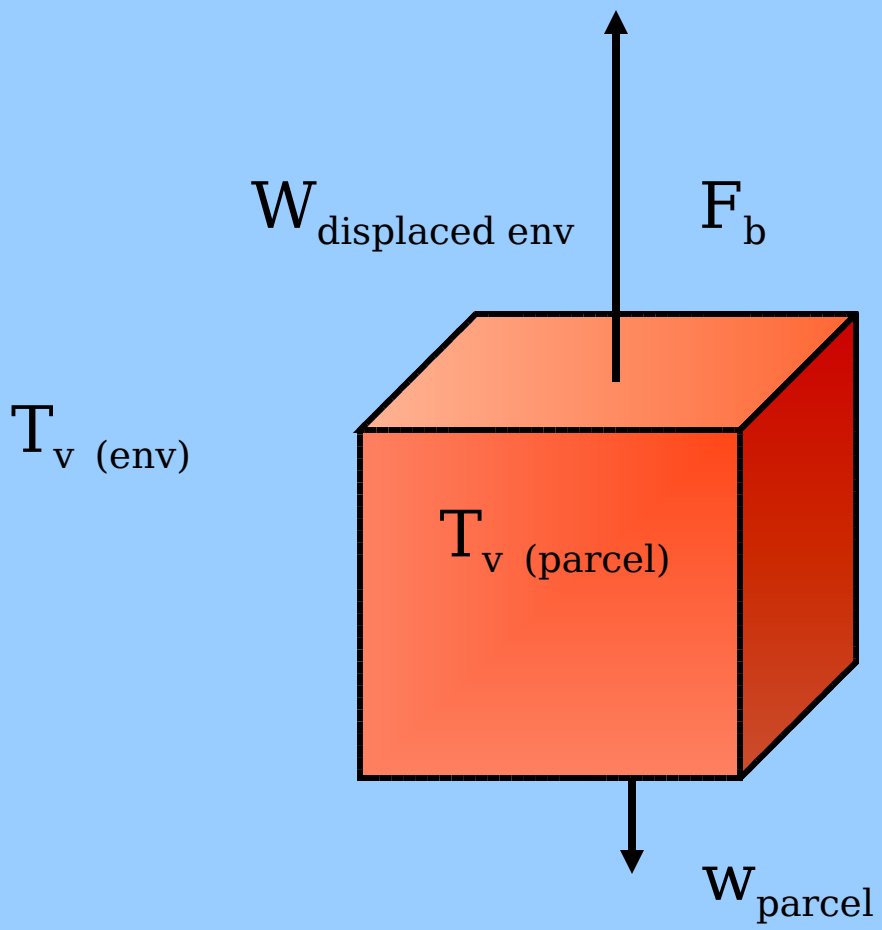
- The buoyant force of the atmosphere surrounding the parcel is directly proportional to the density of the atmosphere.
- Review of density dependence on temperature and moisture content (using the Equation of State $P = \rho R T$)
  - For a moist parcel of air, use virtual temperature to take into account the moisture in the atmosphere $T_v = T + w/6$.
  - At constant pressure, density is inversely proportional to the virtual temperature. $P = \rho R T_v$

Atmospheric Stability

- A displaced parcel of air will move vertically in a direction determined by the strength of the buoyant force.
- The strength of the buoyant force is determined by comparing the environmental T_v (i.e. density) with the parcel T_v.

Atmospheric Stability

- If T_v of the parcel is greater than Tv of the environment:
  - The density of the parcel is less than the environment. (inversely proportional)
  - The weight of the parcel is less than the weight of the displaced atmosphere.
  - The buoyant force is stronger than the weight of the parcel.
  - The parcel accelerates upward.



$T_{v\ (parcel)} > T_{v\ (env)}$

ρ is inversely proportional to T_v

$\rho_{parcel} < \rho_{env}$

Weight is directly proportional to ρ

$w_{parcel} < w_{displaced\ env}$

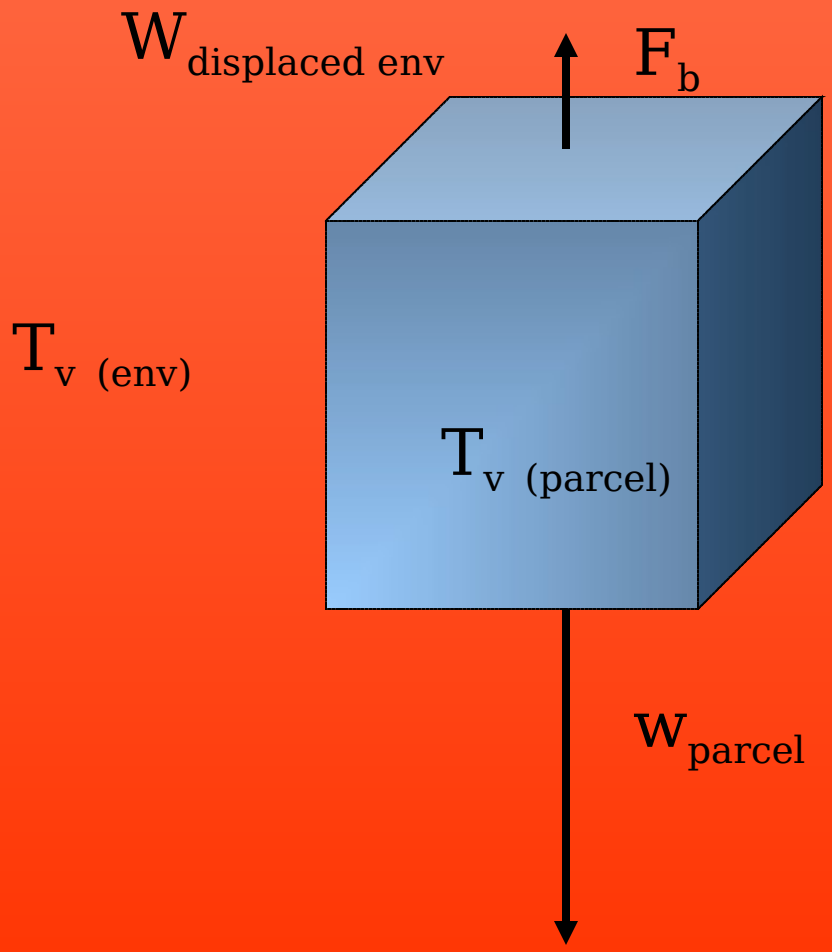
$w_{displaced\ env} = F_b$

$w_{parcel} < F_b$

Hence....
The parcel accelerates UPWARD.

Atmospheric Stability

- If T_v of the parcel is lower than Tv of the environment
  - The density of the parcel is greater than the environment (inversely proportional).
  - The weight of the parcel is greater than the weight of the displaced atmosphere.
  - The buoyant force is weaker than the weight of the parcel.
  - The parcel accelerates downward.



$T_{v-(parcel)} < T_{v\ (env)}$

ρ is inversely proportional to T_v

$\rho_{parcel} > \rho_{env}$

Weight is directly proportional to ρ

$w_{parcel} > w_{displaced\ env}$

$w_{displaced\ env} = F_b$

$w_{parcel} > F_b$

Hence....
The parcel accelerates DOWNWARD.

Atmospheric Stability

- **Stability Classifications**
  - **Stable** - a displaced parcel of air will return to its original position. Vertical motion is suppressed (includes both upward and downward motion).

    $(w > F_b)$, $(\rho_{parcel} > \rho_{environment})$, and

    $(T_{v(parcel)} < T_{v(environment)})$
  - **Unstable** - a displaced parcel will continue in the direction displaced, vertical motion is enhanced (both upward and downward).

    $(w < F_b)$, $(\rho_{parcel} < \rho_{environment})$, and

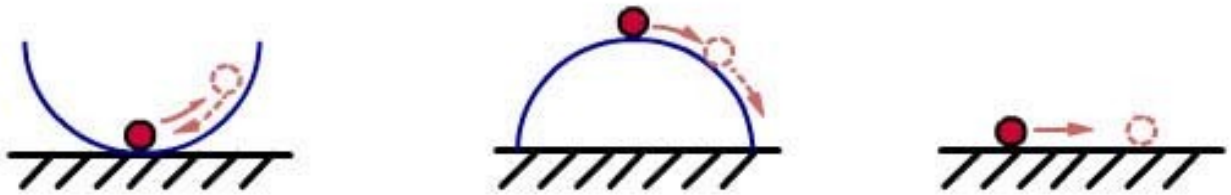
    $(T_{v(parcel)} > T_{v(environment)})$

Atmospheric Stability

- **Stability Classifications cont.**
  - **Neutral** - a displaced parcel will remain in the position to which it was displaced, vertical motion is neither enhanced nor suppressed ('neutral' effect).

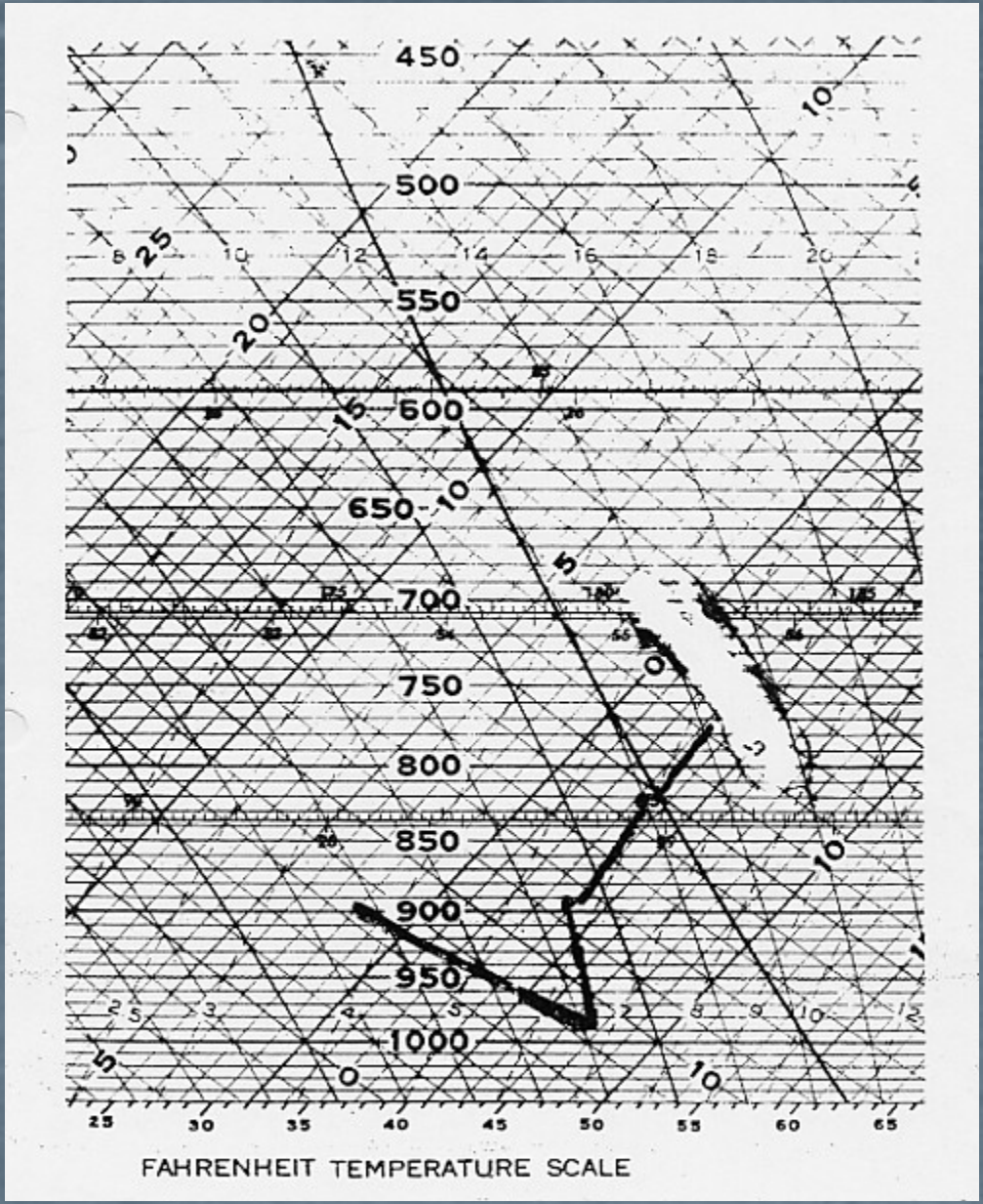
  $(w = F_b)$, $(\rho_{parcel} = \rho_{environment})$, and $(T_{v(parcel)} = T_{v(environment)})$

Stability Classifications



Atmospheric Stability

- **Determining Stability from Lapse Rate**
  - Lapse rate is the change in temperature with height.
  - $\gamma = -(\Delta T / \Delta Z) = -[(T_{upper} - T_{lower}) / (Z_{upper} - Z_{lower})]$
    - The environmental lapse rate (γ_e) is determined by the atmospheric sounding.
    - The lapse rate at which the parcel changes temperature is determined by the moisture content of the parcel.
      - Unsaturated parcels change according to the dry adiabatic rate (γ_d) .
      - Saturated parcels change according to the saturation (moist) adiabatic rate (γ_m) .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FAHRENHEIT TEMPERATURE SCALE

Atmospheric Stability

- **Determining Stability from Lapse Rate cont.**
  - Comparisons between the environmental and the parcel lapse rates determine the atmospheric stability. (The parcel may be lifted using either the moist or the dry adiabatic lapse rate, depending on it's moisture content.)
    - If a rising parcel (moist or dry) cools faster than the environmental temperature (the parcel lapse rate is greater than the environmental lapse rate), the atmosphere is STABLE.

$$\gamma_d > \gamma_m > \gamma_e$$

Atmospheric Stability

- If a rising parcel cools more slowly than the environmental temperature (the parcel lapse rate is less than the environmental lapse rate), the atmosphere is UNSTABLE.

$$\gamma_e > \gamma_d > \gamma_m$$

- If a rising parcel cools at the same rate as the environmental temperature (the parcel lapse rate is the same as the environmental lapse rate), the atmosphere has NEUTRAL STABILITY.

$\gamma_m = \gamma_e$, **if the parcel is saturated**

$\gamma_d = \gamma_e$, **if the parcel is unsaturated**

Atmospheric Stability

- If the environmental temperature decreases with height at a rate less than that of an unsaturated parcel, and at a rate greater than that of a saturated parcel, the stability is dependent on the moisture characteristics of the parcel. The atmosphere has CONDITIONAL STABILITY.

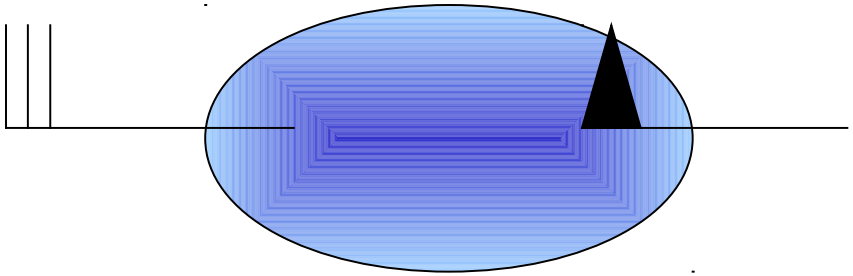
$$\gamma_d > \gamma_e > \gamma_m$$

Mass Continuity Theory

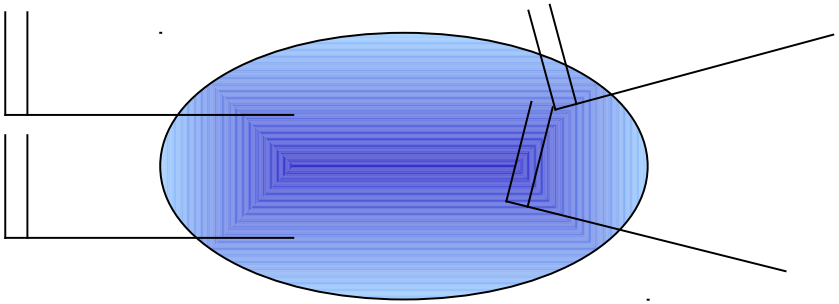
- Divergence
  - Air leaves a region faster than it enters due to differences in wind velocities (speed, direction, or both). This covered in greater detail in Dynamics I.
  - Causes a net loss of mass in the region.

Divergence

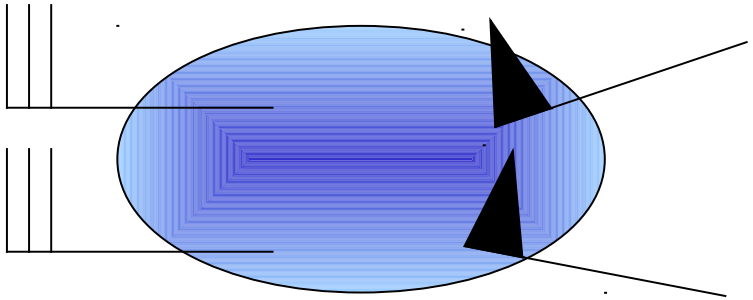
Difference in speed



Difference in direction



Differences in speed and direction

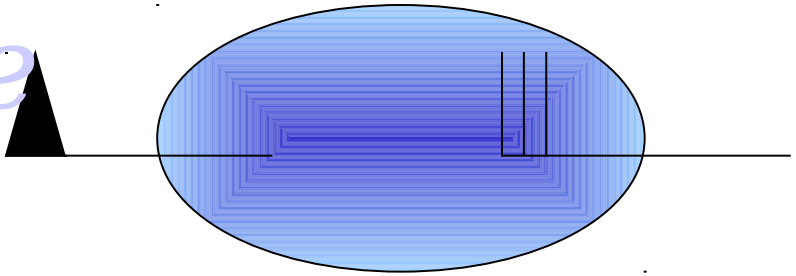


Mass Continuity Theory

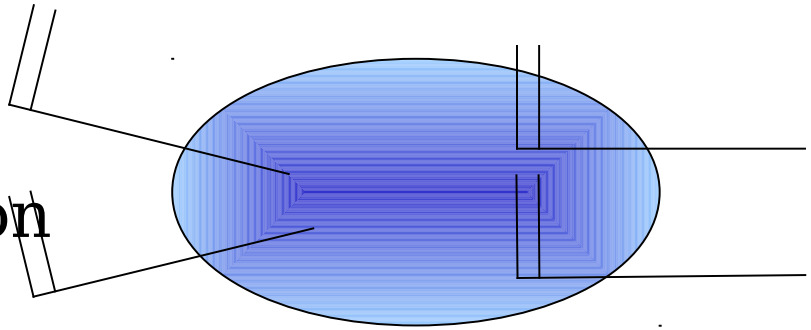
- Convergence
  - Air enters an area faster than it leaves due to differences in wind velocities (speed, direction, or both).
  - Causes a net gain of mass in the region.

Convergence

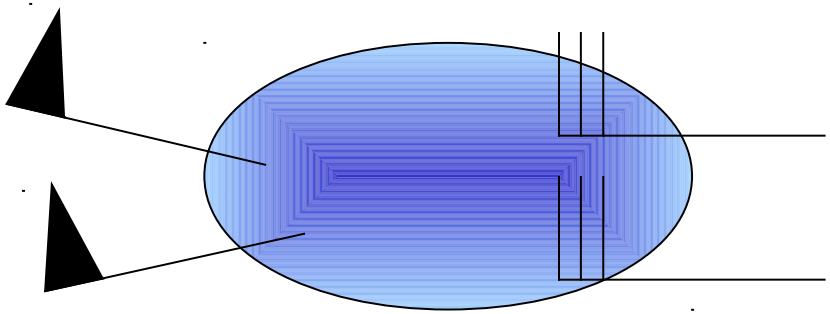
Difference in speed



Difference in direction



Differences in speed and direction



Mass Continuity Theory

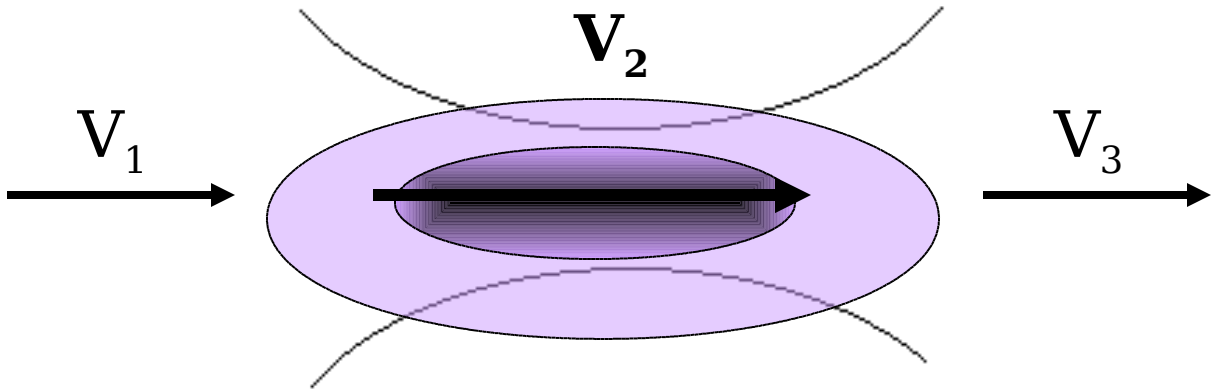
- Continuity Equation
  - The net gain or loss of mass in a system is equal to the amount of mass entering the system minus the amount of mass leaving the system.
    - Therefore there is no net gain or loss of mass in the system.
    - The amount of mass remains constant. (The pressure remains constant.)
    - The amount of mass entering the system equals the amount of mass leaving the system.

Mass Continuity Theory

- Continuity Equation
  - Two horizontal dimensions: The amount of mass entering a region must equal the amount leaving. The amount of mass in a region remains constant
    - The horizontal divergence must counteract the horizontal convergence in the region.
    - The horizontal convergence must balance the horizontal divergence in the region.
    - In this balanced mode, the net effect is that the total divergence within the system is zero.

CONTINUITY EQUATION (2-D)

As the channel narrows, the velocity must increas

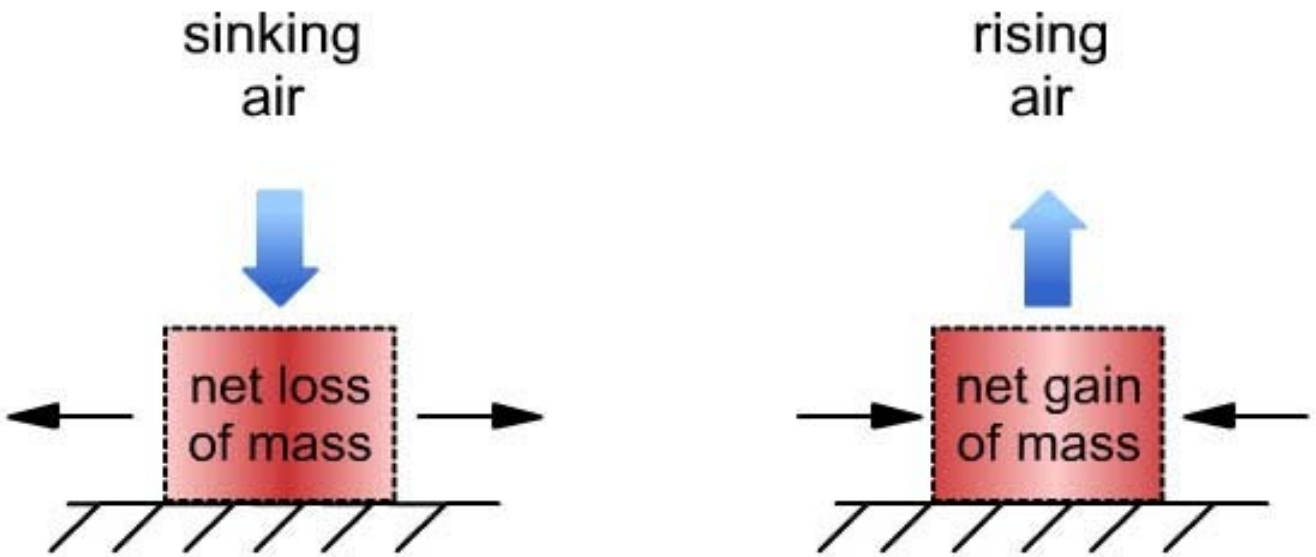


As the channel widens, the velocity must decreas

Mass Continuity Theory

- Continuity Equation in 3 dimensions: Adds vertical motion to the two dimensional version.
  - If there is an area of net divergence at the surface, air must sink into the area to replace the net loss of mass.
  - If there is an area of net convergence at the surface, air must flow upward out of the area to expel the net gain of mass.

Continuity Equation (3-D)



Horizontal divergence Horizontal convergence

Note: Arrows indicate direction of flow only, not magnitude.

Energy

- Kinetic Energy (KE) - energy of motion
  - Equation: $KE = 1/2\ m\ V^2$
    - m is the mass
    - V is the velocity
  - Units: Joule (J) = N m = kg m^2/s^2

Energy

- Variable relationships
  - Kinetic Energy is directly proportional to mass.

$$\uparrow KE = 1/2 \uparrow m\ V^2$$

    - Assume the velocity is held constant.
    - If the mass is increased, the kinetic energy is increased.
    - If the mass is decreased, the kinetic energy is decreased

Energy

- Kinetic Energy is directly proportional to the square of the velocity.

$$\uparrow KE = 1/2\, m\, V^2 \qquad \uparrow V^2$$

  - Assume the mass is held constant.
  - If the velocity is increased, the kinetic energy is increased by that amount squared.
  - If the velocity is decreased, the kinetic energy is decreased by that amount squared.

$$\downarrow KE = 1/2\, m\, V^2 \qquad \downarrow V^2$$

Energy

- Potential Energy (PE) - energy due to position
  - Equation: PE = m **g** h
    - m is the mass
    - **g** is the acceleration due to gravity (9.8 m/s^2)
    - h is the height above the ground
  - Units: Joule (J) - same as kinetic energy

Energy

- Potential Energy
  - Variable Relationships
    - Potential Energy is directly proportional to mass.
      - Assume the height is held constant (g is constant by definition).
      - If the mass is increased, the potential energy in increased.

$$\uparrow PE = \uparrow m\ g\ h$$

      - If the mass is decreased, the potential energy in decreased.

$$\downarrow PE = \downarrow m\ g\ h$$

Energy

- Potential Energy
  - Potential Energy is directly proportional to height.
    - Assume the mass is held constant (g is constant by definition).
    - If the height is increased, the potential energy in increased.

m

↑ PE = g ↑ h

    - If the height is decreased, the potential energy in decreased.

m

↓ PE = g ↓ h

Energy

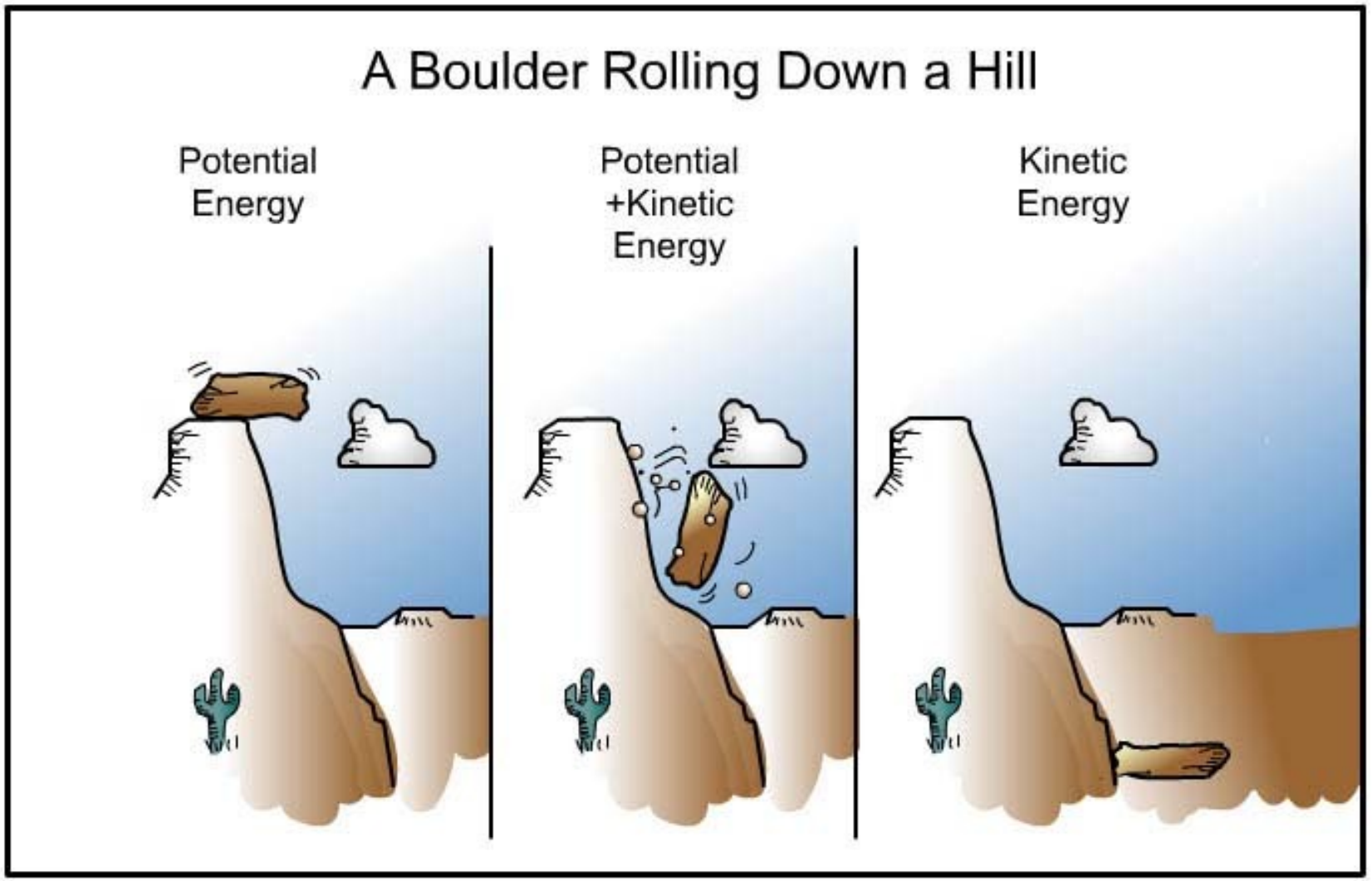
- Conservation of Energy
  - The total energy of a system ($\Delta KE + \Delta PE$) remains constant.
  - Any change in kinetic energy will result in a equal change in potential energy.

$$\Delta KE = - \Delta PE$$

    - A decrease in KE will result in an increase in PE. KE is CONVERTED to PE.
    - An increase in KE will result in a decrease in PE. PE is CONVERTED to KE.

Energy

- Example 1.
  - **A boulder rolling down a hill**
    - Initially at the top of the hill, the boulder is at rest. All the energy it possesses is potential energy.
    - As the boulder starts to roll down the hill, the potential energy decreases because the height above the base of the hill has decreased. (PE = m**g**h)
    - The total energy of the boulder is a combination of potential and kinetic energy.
    - As it moves further down the hill, more PE is converted to KE.


A Boulder Rolling Down a Hill
Potential Energy
Potential +Kinetic Energy
Kinetic Energy

Energy

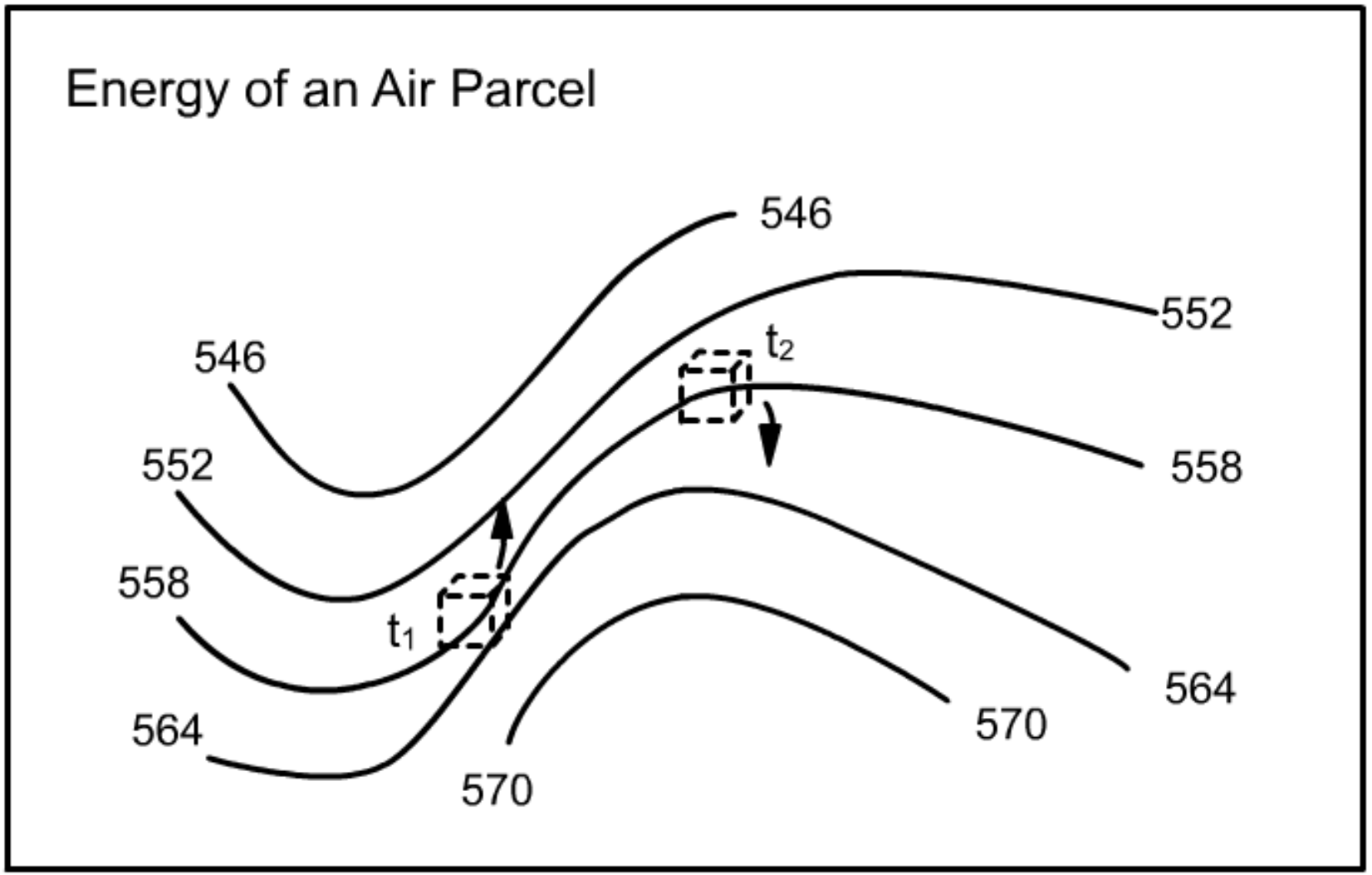
- Example 1
  - **A boulder rolling down a hill**
    - The velocity of the boulder increases.

      $KE = 1/2\ m\ V^2$.
    - When the boulder reaches the bottom of the hill, its potential energy is equal to zero (h = 0).
    - The energy is now all kinetic and the velocity is at its maximum.

    (Note: The above discussion ignores frictional influences.)

Energy

- Example 2
  - An air parcel is at 558 dam on the 500mb surface. The parcel remains at 500mb, but is deflected to its left (toward lower heights).
    - What happens to its PE?
    - What happens to its KE?
    - What happens to the velocity of the air parcel?
    - What is the relationship to the magnitude of each change?


Energy of an Air Parcel
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Energy

- Example 3
  - Later, the same parcel is deflected to the right. How do the two situations compare?
    - A deflection toward the right means toward higher heights (Buys-Ballots' Law).
    - An increase in height corresponds to an increase in potential energy.
    - An increase in potential energy corresponds to an equal decrease in kinetic energy.
    - The temperature of a parcel of air is determined by the kinetic energy of the molecules making up the parcel.

Energy

- Later, the same parcel is deflected to the right.
  - Latent heat is a form of potential energy.
  - If water vapor is present in the air, latent heat is available to be released either during condensation or deposition.
  - The released latent heat can be used to heat the atmosphere causing warmer air and enhanced vertical motion (air which is warmer than its environment rises).

Energy

- The temperature of a parcel of air is determined by the kinetic energy of the molecules making up the parcel.
- Latent heat is a form of potential energy.
  - If water vapor is present in the air, latent heat is available to be released either during condensation or deposition.
  - The released latent heat can be used to heat the atmosphere causing warmer air and enhanced vertical motion (air which is warmer than its environment rises).

- APPLICATION: Assign and review Exercise in WB-100
- EVALUATION: Administer and evaluate Appraisal 102f. Give written test at the end of the unit.